< 정답 및 해설 >

<정답>

문항	1	2	3	4	5	6	7	8	9	10	11	12	13
정답	③	②	②	④	③	④	②	④	①	③	②	②	④
문항	14	15	16	17	18	19	20	21	22	23	24	25	
정답	②	①	③	④	③	③	①	②	①	④	①	①	

<해설>

1. $2x-y-xy'+2yy'=0,\ (x-2y)y'=2x-y,\ y'=\dfrac{2x-y}{x-2y}$

   $(1,1)$에서 $y'=\dfrac{1}{-1}=-1$이므로 접선의 방정식은

   $y-1=-1(x-1),\ y=-x+2$

2. $f(0)=5+0+e^0=6$에서 $f^{-1}(6)=0,\ f'(x)=2+e^x$에서 $f'(0)=3$이므로

   $(f^{-1})'(6)=\dfrac{1}{f'(f^{-1}(6))}=\dfrac{1}{f'(0)}=\dfrac{1}{3}$

3. $g(x)=\displaystyle\int_{\sin x}^{x^3}\frac{1}{\sqrt{1+t^2}}\,dt$

   $=\displaystyle\int_{\sin x}^{1}\frac{1}{\sqrt{1+t^2}}\,dt+\int_{1}^{x^3}\frac{1}{\sqrt{1+t^2}}\,dt$

   $=\displaystyle-\int_{1}^{\sin x}\frac{1}{\sqrt{1+t^2}}\,dt+\int_{1}^{x^3}\frac{1}{\sqrt{1+t^2}}\,dt$

   $g'(x)=-\dfrac{1}{\sqrt{1+\sin^2 x}}\dfrac{d}{dx}(\sin x)+\dfrac{1}{\sqrt{1+x^6}}\dfrac{d}{dx}(x^3)=-\dfrac{\cos x}{\sqrt{1+\sin^2 x}}+\dfrac{3x^2}{\sqrt{1+x^6}}$

   $g'(0)=-1$

4. $V=\int_{-1}^{1}2\pi(y+2)\left[(y^2+2)-3y^2\right]dy=4\pi\int_{-1}^{1}(y+2)(1-y^2)\,dy$

$=4\pi\int_{-1}^{1}(-y^3-2y^2+y+2)\,dy=8\pi\int_{0}^{1}(-2y^2+2)\,dy$

$=16\pi\int_{0}^{1}(1-y^2)\,dy=16\pi\left[y-\frac{1}{3}y^3\right]_0^1=\frac{32}{3}\pi$

5. $x=2\sin\theta$, $dx=2\cos\theta\,d\theta$, $x=0$ 일 때 $\theta=0$, $x=1$ 일 때 $\theta=\frac{\pi}{6}$

$\int_0^1\frac{x^2}{\sqrt{4-x^2}}\,dx=\int_0^{\frac{\pi}{6}}\frac{4\sin^2\theta}{\sqrt{4(1-\sin^2\theta)}}\,2\cos\theta\,d\theta=4\int_0^{\frac{\pi}{6}}\sin^2\theta\,d\theta$

$=4\int_0^{\frac{\pi}{6}}\frac{1}{2}(1-\cos2\theta)\,d\theta=2\left[\theta-\frac{1}{2}\sin2\theta\right]_0^{\frac{\pi}{6}}=\frac{\pi}{3}-\frac{\sqrt{3}}{2}$

6. $y=\frac{x^2}{2}-\frac{1}{4}\ln x$, $y'=x-\frac{1}{4x}$

$L=\int_1^e\sqrt{1+(y')^2}\,dx=\int_1^e\sqrt{x^2+\frac{1}{2}+\left(\frac{1}{4x}\right)^2}\,dx$

$=\int_1^e\left(x+\frac{1}{4x}\right)dx=\left[\frac{x^2}{2}+\frac{1}{4}\ln x\right]_1^e=\frac{e^2}{2}-\frac{1}{4}$

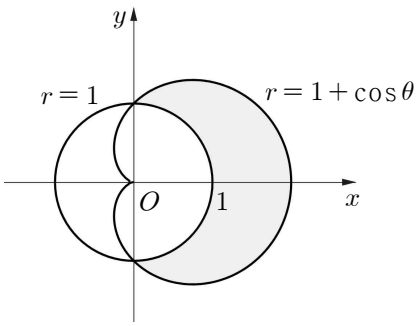
7. $1+\cos\theta=1$, $\cos\theta=0$에서 $\theta=-\frac{\pi}{2}$ 또는 $\theta=\frac{\pi}{2}$

$A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{2}[(1+\cos\theta)^2-1^2]\,d\theta$

$=2\int_0^{\frac{\pi}{2}}\frac{1}{2}(\cos^2\theta+2\cos\theta)\,d\theta$

$=\int_0^{\frac{\pi}{2}}\left(\frac{1+\cos2\theta}{2}+2\cos\theta\right)d\theta$

$=\left[\frac{\theta}{2}+\frac{1}{4}\sin2\theta+2\sin\theta\right]_0^{\frac{\pi}{2}}=\frac{\pi}{4}+2$



8. 평행사변형은 벡터 $a=\overrightarrow{OP}=\langle2,3,1\rangle$, $b=\overrightarrow{OR}=\langle4,2,5\rangle$에 의해 결정된다.

$a\times b=\begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 4 & 2 & 5 \end{vmatrix}=13i-6j-8k$

이므로 평행사변형의 넓이는 $|a\times b|=\sqrt{269}$

9. 곡선의 벡터방정식 $r(t)=\langle 1+2\sqrt{t},\ t^3-t,\ t^3+t\rangle$에서

$r'(t)=\left\langle \dfrac{1}{\sqrt{t}},\ 3t^2-1,\ 3t^2+1\right\rangle$

점 $(3,0,2)$는 $t=1$에 대응하므로 점 $(3,0,2)$에서의 접선벡터는 $r'(1)=\langle 1,2,4\rangle$

따라서 접선의 방정식은

$x=3+t,\ y=2t,\ z=2+4t$

10. $y'=\dfrac{1}{x},\ y''=-\dfrac{1}{x^2}$

$\kappa(x)=\dfrac{|y''|}{\left[1+(y')^2\right]^{3/2}}=\left|-\dfrac{1}{x^2}\right|\dfrac{1}{(1+1/x^2)^{3/2}}=\dfrac{1}{x^2}\dfrac{(x^2)^{3/2}}{(x^2+1)^{3/2}}=\dfrac{x}{(x^2+1)^{3/2}}$

에서 $\kappa(1)=\dfrac{1}{2\sqrt{2}}$

11. $\nabla f(x,y,z)=\left\langle \dfrac{2}{z},\ -\dfrac{1}{z},\ -\dfrac{2x-y}{z^2}\right\rangle$ 에서 $\nabla f(1,1,1)=\langle 2,\ -1,\ -1\rangle$이므로

최대 변화율은 $|\nabla f(1,1,1)|=\sqrt{4+1+1}=\sqrt{6}$

12. $\nabla f(1,2)=\langle f_x(1,2),\ f_y(1,2)\rangle\ =\langle 2,21\rangle$

$v=\langle 6,8\rangle$ 방향의 단위벡터는 $u=\left\langle \dfrac{3}{5},\dfrac{4}{5}\right\rangle$

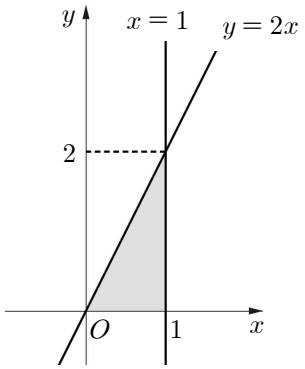
$D_u f(1,2)=\nabla f(1,2)\cdot u=\langle 2,21\rangle\cdot\left\langle \dfrac{3}{5},\dfrac{4}{5}\right\rangle=\dfrac{6}{5}+\dfrac{84}{5}=\dfrac{90}{5}=18$

13. $\displaystyle\int_0^2\int_{\frac{y}{2}}^1 ye^{x^3}dx\,dy=\int_0^1\int_0^{2x} ye^{x^3}dy\,dx$

$\displaystyle=\int_0^1\left[\frac{y^2}{2}e^{x^3}\right]_{y=0}^{y=2x}dx$

$\displaystyle=\int_0^1 2x^2e^{x^3}dx$

$\displaystyle=\frac{2}{3}\left[e^{x^3}\right]_0^1=\frac{2}{3}(e-1)$



14. $P=2x+e^y,\ Q=2y+xe^y$라 하면 $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$이므로 포텐셜함수 ϕ가 존재한다.

$\phi(x,y)=\int Pdx+g(y)=x^2+xe^y+g(y)$

$\frac{\partial \phi}{\partial y}=Q$에서 $g'(y)=2y$이므로 $\phi(x,y)=x^2+xe^y+y^2+K$ (K는 상수이다.)

$\int_C (2x+e^y)dx+(2y+xe^y)dy=\phi(1,1)-\phi(0,0)=e+2$

15. $A=\begin{pmatrix}2 & -3 & 3\\ 0 & -1 & 3\\ 1 & -1 & 1\end{pmatrix}$라 하면

$\det(A-\lambda I)=\begin{vmatrix}2-\lambda & -3 & 3\\ 0 & -1-\lambda & 3\\ 1 & -1 & 1-\lambda\end{vmatrix}=-\lambda^3+2\lambda^2+\lambda-2=-(\lambda+1)(\lambda-1)(\lambda-2)=0$

에서 행렬 A의 고윳값은 $-1,\ 1,\ 2$

16. $M=xy^3-x,\ N=kx^2y^2-\sin y$라 하면 $M_y=3xy^2,\ N_x=2kxy^2$이므로

$M_y=N_x$에서 $k=\frac{3}{2}$

17. $x\,dy=y(x+2)\,dx,\ \frac{1}{y}\,dy=\frac{x+2}{x}\,dx=\left(1+\frac{2}{x}\right)dx$

$\ln|y|=x+2\ln|x|+c_1,\ y(x)=cx^2e^x$

$y(1)=2$에서 $c=\frac{2}{e}$ 이므로 $y(x)=2x^2e^{x-1},\ y(2)=8e$

18. $yy'+y^2=x$

$u=y^2$이라 하면 $u'=2yy'$이므로 $\frac{1}{2}u'+u=x,\ u'+2u=2x$

적분인자는 $e^{\int 2dx}=e^{2x}$이고 $\frac{d}{dx}(e^{2x}u)=2xe^{2x}$에서

$e^{2x}u=xe^{2x}-\frac{1}{2}e^{2x}+c,\ u=\{y(x)\}^2=x-\frac{1}{2}+ce^{-2x}$

$y(0)=1$에서 $c=\frac{3}{2},\ \{y(x)\}^2=x-\frac{1}{2}+\frac{3}{2}e^{-2x}$

따라서 $\{y(1)\}^2=\frac{1+3e^{-2}}{2}$

19. $y''-2y'+y=0$의 보조방정식은 $m^2-2m+1=(m-1)^2=0$이므로 일차독립인 해는 $y_1=e^x,\ y_2=xe^x$

$$W(y_1, y_2)=\begin{vmatrix} e^x & xe^x \\ e^x & (x+1)e^x \end{vmatrix}=e^{2x}$$

$$W_1=\begin{vmatrix} 0 & xe^x \\ \dfrac{e^x}{x^2+1} & (x+1)e^x \end{vmatrix}=-\frac{xe^{2x}}{x^2+1},\quad W_2=\begin{vmatrix} e^x & 0 \\ e^x & \dfrac{e^x}{x^2+1} \end{vmatrix}=\frac{e^{2x}}{x^2+1}$$

$u'=\dfrac{W_1}{W}=-\dfrac{x}{x^2+1},\ v'=\dfrac{W_2}{W}=\dfrac{1}{x^2+1}$ 이므로 $u=-\dfrac{1}{2}\ln(x^2+1),\ v=\tan^{-1}x$

따라서 $y_p=uy_1+vy_2=-\dfrac{e^x}{2}\ln(x^2+1)+xe^x\tan^{-1}x$

20. 보조방정식 $m^2-2m+2=0$에서 $m=1+i$ 또는 $m=1-i$
해는 $y(x)=x(a\cos(\ln x)+b\sin(\ln x))$ 이므로 $p=1,\ q=1$
$y(1)=a=2$
$y'=(a\cos(\ln x)+b\sin(\ln x))+(-a\sin(\ln x)+b\cos(\ln x))$
$y'(1)=a+b=0$에서 $b=-2$
따라서 $a+b+p+q=2$

21. $f(t)=\mathcal{L}^{-1}\left\{\dfrac{1}{s(s^2+4)}\right\}=\mathcal{L}^{-1}\left\{\dfrac{1}{4}\left(\dfrac{1}{s}-\dfrac{s}{s^2+4}\right)\right\}=\dfrac{1}{4}(1-\cos 2t)$이므로 $f\left(\dfrac{\pi}{4}\right)=\dfrac{1}{4}$

22. $f(t)=\cos 2t-\cos 2t\ u\left(t-\dfrac{\pi}{4}\right)$

$$\mathcal{L}\{f(t)\}=\mathcal{L}\{\cos 2t\}-\mathcal{L}\left\{\cos 2t\,u\left(t-\frac{\pi}{4}\right)\right\}=\frac{s}{s^2+4}-\mathcal{L}\left\{\cos 2\left(t+\frac{\pi}{4}\right)\right\}e^{-\frac{\pi}{4}s}$$

$$=\frac{s}{s^2+4}-\mathcal{L}\left\{\cos\left(2t+\frac{\pi}{2}\right)\right\}e^{-\frac{\pi}{4}s}=\frac{s}{s^2+4}+\mathcal{L}\{\sin 2t\}e^{-\frac{\pi}{4}s}$$

$$=\frac{s}{s^2+4}+\frac{2}{s^2+4}e^{-\frac{\pi}{4}s}$$

23. $y(t)=t^2+e^{-t}*y(t)$ ($e^{-t}*y(t)$는 e^{-t}와 $y(t)$의 합성곱이다.)

$\mathcal{L}\{y(t)\}=Y(s)$라 하면 $Y(s)=\frac{2}{s^3}+\frac{1}{s+1}Y(s)$, $\frac{s}{s+1}Y(s)=\frac{2}{s^3}$ 에서

$Y(s)=\frac{2}{s^4}(s+1)=\frac{2}{s^3}+\frac{2}{s^4}$

$y(t)=\mathcal{L}^{-1}\{Y(s)\}=t^2+\frac{t^3}{3}$이므로 $y(1)=\frac{4}{3}$

24. $F(s)=\int_0^\infty e^{-sx}x\cos 2x\,dx$라 하면

$F(s)=\mathcal{L}\{x\cos 2x\}=-\frac{d}{ds}\mathcal{L}\{\cos 2x\}=\frac{s^2-4}{(s^2+4)^2}$

따라서 $\int_0^\infty e^{-3x}x\cos 2x\,dx=F(3)=\frac{5}{169}$

25. $t=x^2$이라 하면 $dt=2x\,dx$

$$\int_0^\infty x^4e^{-x^2}dx=\frac{1}{2}\int_0^\infty t^{\frac{3}{2}}e^{-t}dt$$

$$=\frac{1}{2}\Gamma\left(\frac{5}{2}\right)=\frac{1}{2}\times\frac{3}{2}\Gamma\left(\frac{3}{2}\right)=\frac{1}{2}\times\frac{3}{2}\times\frac{1}{2}\Gamma\left(\frac{1}{2}\right)=\frac{3}{8}\sqrt{\pi}$$